10.4 Sketching the Nyquist Diagram

The contour that encloses the right half-plane can be mapped through the function $G(s)H(s)$ by substituting points along the contour into $G(s)H(s)$. The points along the positive extension of the imaginary axis yield the polar frequency response of $G(s)H(s)$. Approximations can be made to $G(s)$ for points around the infinite semicircle by assuming that the vectors originate at the origin. Thus their length is infinite, and their angles are easily evaluated.

However, most of the time, a simple sketch of the Nyquist diagram is all that is needed. A sketch can be obtained rapidly by looking at the vectors of $G(s)H(s)$ and their motion along the contour. In the examples that follow, we stress this rapid method for sketching the Nyquist diagram. However, the examples also include analytical expressions for $G(s)H(s)$ for each section of the contour to aid you in determining the shape of the Nyquist diagram.

Example 10.4

Sketching a Nyquist diagram

**Problem** Speed controls find wide application throughout industry and the home. Figure 10.26(a) shows one application: output frequency control of electrical power from a turbine and generator pair. By regulating the speed, the control system ensures that the generated frequency remains within tolerance. Deviations from the desired speed are sensed, and a steam valve is changed to compensate for the speed error. The system block diagram is shown in Figure 10.26(b). Sketch the Nyquist diagram for the system of Figure 10.26.

**Solution** Conceptually, the Nyquist diagram is plotted by substituting the points of the contour shown in Figure 10.27(a) into $G(s) = 500/[(s + 1)(s + 3)(s + 10)]$. This process is equivalent to performing complex arithmetic using the vectors of $G(s)$ drawn to the points of the contour as shown in Figure 10.27(a) and (b). Each pole and zero term of $G(s)$ shown in Figure 10.26(b) is a vector in Figure 10.27(a) and (b). The resultant vector, $R$, found at any point along the contour is in general the product of the zero vectors divided by the product of the pole vectors (see Figure 10.27(c)). Thus, the magnitude of the resultant is the product of the zero lengths divided by the product of the pole lengths, and the angle of the resultant is the sum of the zero angles minus the sum of the pole angles.
At zero frequency, \( G(j\omega) = 500/30 = 50/3 \). Thus, the Nyquist diagram starts at 50/3 at an angle of 0°. As \( \omega \) increases, the real part remains positive, and the imaginary part remains negative. At \( \omega = \sqrt{30/14} \), the real part becomes negative. At \( \omega = \sqrt{43} \), the Nyquist diagram crosses the negative real axis since the imaginary term goes to zero. The real value at the axis crossing, point \( Q' \) in Figure 10.27(c), found by substituting \( \omega = \sqrt{43} \) into Eq. (10.41), is \(-0.874\). Continuing toward \( \omega = \infty \), the real part is negative, and the imaginary part is positive. At infinite frequency, \( G(j\omega) \approx 500j/\omega^3 \), or approximately zero at 90°.

Around the infinite semicircle from point \( C \) to point \( D \), shown in Figure 10.27(b), the vectors rotate clockwise, each by 180°. Hence, the resultant undergoes a counterclockwise rotation of 3 \( \times \) 180°, starting at point \( C' \) and ending at point \( D' \) of Figure 10.27(c). Analytically, we can see this by assuming that around
As we move in a clockwise direction around the contour from point A to point C in Figure 10.27(a), the resultant angle goes from $0^\circ$ to $-3 \times 90^\circ = -270^\circ$, or from $A'$ to $C'$ in Figure 10.27(c). Since the angles emanate from poles in the denominator of $G(s)$, the rotation or increase in angle is really a decrease in angle of the function $G(s)$; the poles gain $270^\circ$ in a counterclockwise direction, which explains why the function loses $270^\circ$.

While the resultant moves from $A'$ to $C'$ in Figure 10.27(c), its magnitude changes as the product of the zero lengths divided by the product of the pole lengths. Thus, the resultant goes from a finite value at zero frequency (at point A of Figure 10.27(a) there are three finite pole lengths) to zero magnitude at infinite frequency at point C (at point C of Figure 10.27(a) there are three infinite pole lengths).

The mapping from point A to point C can also be explained analytically. From $A$ to $C$ the collection of points along the contour is imaginary. Hence, from $A$ to $C$, $G(s) = G(j\omega)$, or from Figure 10.26(b),

$$G(j\omega) = \frac{500}{(s+1)(s+3)(s+10)} \bigg|_{s \to j\omega} = \frac{500}{(-14\omega^2 + 30) + j(43\omega - \omega^3)}$$

(10.40)

Multiplying the numerator and denominator by the complex conjugate of the denominator, we obtain

$$G(j\omega) = \frac{500(-14\omega^2 + 30) - j(43\omega - \omega^3)}{(-14\omega^2 + 30)^2 + (43\omega - \omega^3)^2}$$

(10.41)